

Nr. 9 a) $\frac{x^2+6}{3x} - \frac{x-1}{x} = \frac{4x-1}{3x} \rightarrow D = \mathbb{R} \setminus \{0\} \quad \text{HN: } 3x$

$$\frac{x^2+6}{3x} \cdot 3x - \frac{(x-1)}{x} \cdot 3x = \frac{4x-1}{3x} \cdot 3x$$

$$x^2+6 - (3x-3) = 4x-1$$

$$x^2+6 - 3x+3 = 4x-1 \quad | -4x+1$$

$$x^2 - 7x + 10$$

$$x_{1/2} = -\frac{-7}{2} \pm \sqrt{\left(\frac{-7}{2}\right)^2 - 10}$$

$$x_1 = \frac{7}{2} + \sqrt{\frac{49}{4} - \frac{40}{4}}$$

$$x_1 = \frac{7}{2} + \frac{3}{2} = 5$$

$$x_2 = \frac{7}{2} - \frac{3}{2} = 2$$

$$L = \{2; 5\}$$

b) $\frac{x+1}{3x} + \frac{3x-1}{5x} = \frac{x^2+26}{15x} \rightarrow D = \mathbb{R} \setminus \{0\} \quad \text{HN: } 15x$

$$\frac{(x+1)}{3x} \cdot \frac{5}{15x} + \frac{(3x-1)}{5x} \cdot \frac{3}{15x} = \frac{x^2+26}{15x} \cdot 15x$$

$$5x+5 + 9x-3 = x^2+26 \quad | -x^2 | -26$$

$$-x^2 + 14x - 24$$

$$| : -1$$

$$x^2 - 14x + 24$$

$$x_{1/2} = -\frac{-14}{2} \pm \sqrt{\left(\frac{-14}{2}\right)^2 - 24}$$

$$= 7 \pm \sqrt{25}$$

$$x_1 = 7 + 5 = 12$$

$$x_2 = 7 - 5 = 2$$

$$L = \{2; 12\}$$

$$c) \frac{x+3}{4x} = \frac{x^2+5}{6x} - \frac{x-1}{3x} \rightarrow D = \mathbb{R} \setminus \{0\} \quad \text{HN: } 12x$$

$$\frac{(x+3)}{4x} \cdot 12x = \frac{(x^2+5)}{6x} \cdot 12x - \frac{(x-1)}{3x} \cdot 12x$$

$$3x+9 = 2x^2+10 - (4x-4)$$

$$3x+9 = 2x^2+10 - 4x+4 \quad | -3x \quad | -9$$

$$0 = 2x^2 - 7x + 5 \quad | : 2$$

$$0 = x^2 - \frac{7}{2}x + \frac{5}{2}$$

$$x_{1/2} = + \frac{7}{4} \pm \sqrt{\left(\frac{7}{4}\right)^2 - \frac{5}{2}} \quad \frac{7}{2} : 2 = \frac{7}{2} \cdot \frac{1}{2} = \left(\frac{7}{4}\right)$$

$$x_{1/2} = \frac{7}{4} \pm \sqrt{\frac{49}{16} - \frac{40}{16}}$$

$$x_1 = \frac{7}{4} + \frac{3}{4} = \frac{10}{4} = 2\frac{1}{2} \quad L = \left\{ 1; 2\frac{1}{2} \right\}$$

$$x_2 = \frac{7}{4} - \frac{3}{4} = \frac{4}{4} = 1$$

$$d) \frac{1}{4x+2} - \frac{x-1}{6x+3} = \frac{x^2-3}{12x+6} \rightarrow D = \mathbb{R} \setminus \left\{ -\frac{1}{2} \right\} \quad \text{HN: } 6(2x+1)$$

Hauptnenner finden:

$$4x+2 \rightarrow \underline{2(2x+1)} \quad ; \quad 6x+3 \rightarrow \underline{3(2x+1)} \quad ; \quad 12x+6 \rightarrow \underline{6(2x+1)}$$

$$\frac{1}{2(2x+1)} \cdot \underline{6(2x+1)} - \frac{(x-1)}{3(2x+1)} \cdot \underline{6(2x+1)} = \frac{x^2-3}{6(2x+1)} \cdot 6(2x+1)$$

$$3 - (2x-2) = x^2-3$$

$$3 - 2x + 2 = x^2 - 3 \quad | +2x \quad | -5$$

$$0 = x^2 + 2x - 8$$

$$x_{1/2} = -\frac{2}{2} \pm \sqrt{\left(\frac{2}{2}\right)^2 + 8}$$

$$x_1 = -1 + 3 = 2 \quad L = \left\{ -4; 2 \right\}$$

$$x_2 = -1 - 3 = -4$$

$$e) \frac{x+1}{2x-2} - \frac{2x^2}{5x-5} = \frac{1-x}{10x-10} \rightarrow D = \mathbb{R} \setminus \{1\} \quad \text{HN: } 10(x-1)$$

Hauptnenner finden:

$$2x-2 \rightarrow \underline{2(x-1)}; \quad 5x-5 \rightarrow \underline{5(x-1)}; \quad 10x-10 \rightarrow \underline{10(x-1)}$$

$$\frac{(x+1)}{2(x-1)} \cdot \frac{5}{10(x-1)} - \frac{2x^2}{5(x-1)} \cdot \frac{2}{10(x-1)} = \frac{(1-x)}{10(x-1)} \cdot 10(x-1)$$

$$5x+5 - 4x^2 = 1-x \quad | +4x^2 \quad | -5x \quad | -5$$

$$4x^2 - 6x - 4 = 0 \quad | :4$$

$$x^2 - \frac{6}{4}x - 1 = 0$$

$$x_{1/2} = \frac{6}{8} \pm \sqrt{\left(\frac{6}{8}\right)^2 + 1}$$

$$x_{1/2} = \frac{6}{8} \pm \sqrt{\frac{36}{64} + \frac{64}{64}}$$

$$x_{1/2} = \frac{6}{8} \pm \sqrt{\frac{100}{64}}$$

$$x_1 = \frac{6}{8} + \frac{10}{8} = \frac{16}{8} = 2$$

$$x_2 = \frac{6}{8} - \frac{10}{8} = -\frac{4}{8} = -\frac{1}{2}$$

$$L = \left\{ -\frac{1}{2}; 2 \right\}$$

f)

$$\frac{x^2+1}{4x+6} + \frac{x^2+2}{6x+9} = \frac{x+7}{8x+12} \rightarrow D = \mathbb{R} \setminus \left\{ -\frac{1}{2}; -\frac{3}{4} \right\} \quad \text{HN: } \frac{12}{4}(2x+3)$$

$$\frac{(x^2+1)}{2(2x+3)} \cdot \frac{6}{6(2x+3)} + \frac{(x^2+2)}{3(2x+3)} \cdot \frac{4}{4(2x+3)} = \frac{(x+7)}{4(2x+3)} \cdot \frac{12}{12(2x+3)}$$

$$6x^2+6 + 4x^2+8 = 3x+21 \quad | -3x \quad | -21$$

$$10x^2 - 3x - 7 = 0 \quad | :10$$

$$x^2 - \frac{3}{10}x - \frac{7}{10}$$

$$x_{1/2} = \frac{3}{20} \pm \sqrt{\left(\frac{3}{20}\right)^2 + \frac{7}{10}}$$

$$x_{1/2} = \frac{3}{20} \pm \sqrt{\frac{9}{400} + \frac{280}{400}}$$

$$x_1 = \frac{3}{20} + \sqrt{\frac{289}{400}}$$

$$x_1 = \frac{3}{20} + \frac{17}{20} = \frac{20}{20} = 1$$

$$x_2 = \frac{3}{20} - \frac{17}{20} = -\frac{14}{20} = -\frac{7}{10}$$

$$L = \left\{ -\frac{7}{10}; 1 \right\}$$